Assessment of the Impact of Layer Moduli on Measured Surface Deflections

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ABSTRACT

This paper deals with the interpretation of falling weight deflectometer test data, with emphasis being placed on presenting a methodology that can be used to determine the sensitivity of measured surface deflections to the location and stiffness of each layer in a pavement/subgrade system. It begins by introducing the notion of contribution ratio and then presents a theory, cast within finite element methodology, that is used to quantify deflection, strain and stress field sensitivity to layer modulus. Although the methodology applies to both elastostatic and dynamic analyses, emphasis is placed on the elastostatic responses. Unlike previous expositions, the authors make direct use of the definition of contribution ratio without having to deal with stochastic finite element modelling. Examples are presented and the authors demonstrate that the results from the analyses are consistent with experience.

INTRODUCTION

The use of an improved subgrade or a subgrade capping layer (SCL) for new highway construction is becoming increasingly popular. For example, the recently constructed Highway 104 Western Alignment in Nova Scotia [*1*] utilizes a subgrade capping layer ranging from sand fill to crushed rock, and the Cross Israel Highway project currently under construction east of Tel Aviv [*2*] incorporates a crushed rock capping layer ranging in thickness from about 20 to 40 cm. Specifications of a typical SCL are provided in Table 1. It should be noted that the specifications usually also include a requirement for in-situ density and moisture content.

Table 1. Typical Properties of Subgrade Capping Layer Material

Criteria	***Specification (%)***
Passing sieve 75.0 mm	100
Passing sieve 19 mm	50-100
Passing sieve 4.75 mm	25-80
Passing sieve 0.075 mm	0-30
Liquid Limit (LL)	<35
Plasticity Index (PI)	<10
CBR	>30

While it is generally agreed that the use of a subgrade capping layer will improve the consistency and overall performance of a pavement structure, there is some disagreement on how to account for the use of a capping layer in design or whether or not the SCL should form part of the subgrade or part of the pavement structure. For example, the American Association of State Highway and Transportation Officials [*3*] suggest that the capping layer is a part of the pavement structure, while the U.S. Federal Aviation Administration [*4*] and British Transportation and Road Research Laboratory [*5*] assess the impact of the capping layer on improving the subgrade support.

Pavement/subgrade structures are complex, non-homogeneous layered systems that comprise materials that are moisture, temperature and stress sensitive, which in turn contribute to the temporal changes in the mechanical properties of the individual pavement layers and therefore the overall structural capacity of a pavement [*6*]. An important measurement tool for evaluating structural capacity is the falling weight deflectometer. In spite of its popularity, a challenge and subject of significant discussion and controversy is the interpretation of test data with regard to determining how the stiffness of each layer in the system influences pavement behavior to surface loading. This paper deals with the interpretation of FWD data, with an emphasis placed on the development of a methodology that can be used to assess the sensitivity of the measured surface deflections, as well as performance parameters such as strains, to the thickness and stiffness of each layer in a pavement and subgrade system.

CONTRIBUTION RATIO

Definition

Given that u_{ik} denotes the deflection contribution of a layer with elastic modulus E_i to the total deflection u_k corresponding to degree of freedom k, the contribution ratio CR_k is defined as

[1] $$CR_k = \frac{u_{ik}}{u_k}.$$

According to this definition $\sum CR_k = 1$, when summed over all k. In other words, the total displacement is obtained by adding up the contributions of u_k from each layer. Given $\mathbf{u}_i = -E_i \mathbf{u}_{,i}$, with $u_{,i}$ denoting differentiation with respect to E_i, a positive contribution ratio implies that an increase in E_i is accompanied by a decrease in u_i. Parvini (1997) showed for stochastic elastostatic finite element analysis, that CR_k is related to the coefficient of variation of u_k and that of E_i by the equation;

[2] $$CR_k = \frac{CV(u_k)}{CV(E_i)}.$$

Eq. 2 indicates that the sensitivity of displacement k to modulus of layer i is related to the statistical uncertainties associated with the modulus of that layer. This property allows one to evaluate the importance of a layer's material stiffness to measured surface deflections. An elastic modulus is highly correlated to a measured displacement if the corresponding CR_k is large, and weakly correlated when the CR_k is small. This provides the engineer with a tool to determine whether a measurement can be realistically used to estimate an in-situ modulus.

It should be noted that situations may arise when employing Eq. 2, in which $\sum CR_k > 1$ since the coefficient of variation of a variable is positive by definition. For this reason the use of Eq. 1 is preferred.

Finite Element Implementation

Strictly speaking, the loading history associated with a falling weight deflectometer test is of an impact nature, thus necessitating the use of elastodynamic analysis when simulating in-situ pavement response to FWD loading. This does not imply that one cannot analyze the significance of layer properties on surface measurements via an elastostatic model. It does mean, however, that backcalculated moduli may contain large errors since important frequency-dependent response information would be missing when neglecting the inertial term. Although the analyses reported later in the paper are based on the elastostatic approximation, the exposition of this section takes into account the dynamic response for purposes of completeness.

Following Stolle and Pavini [*7,8*], let us consider the frequency domain representation of the elastodynamic equilibrium of the pavement-subgrade system, which may be expressed in finite element matrix form as

[3] $$\mathbf{Ku} = \mathbf{F} + \omega^2 \mathbf{Mu}$$

where, $\mathbf{K} = (\mathbf{1} + \mathbf{2}\xi i)K_s$ which depends on the hysteretic damping ratio, ξ, (representing the stiffness matrix) and on $\mathbf{K}_s$ (referring to the static stiffness). **M** is the mass matrix, and **u** and **F** are the complex-valued displacement and load vector corresponding to angular loading frequency (ω). This form of the equation places all forms of loading on the right-hand side with the system stiffness appearing to the left.

The stiffness matrix is decomposed according to

[4] $$\mathbf{K} = \sum_{i=1}^{n} \mathbf{K_i}$$

in which $\mathbf{K}_i$ is the stiffness contribution to the total stiffness **K** of all finite elements having modulus E_i and n is the number of independent moduli (layers). Similarly, the total displacement **u** may be decomposed according to

[5] $$\mathbf{u} = \sum_{i=1}^{n} \mathbf{u_i}$$

with displacement $\mathbf{u}_i$ due to layer ‘i’ being defined as

[6] $$\mathbf{K}\sum_{i=1}^{n} \mathbf{u}_i = \sum_{i=1}^{n} \mathbf{K}_i \mathbf{u} \quad \Rightarrow \quad \mathbf{u}_i = \mathbf{K}^{-1}\mathbf{K}_i \mathbf{u}\,.$$

The u_{ik} and u_k displacement terms appearing in Eq. 1 are obtained by recognizing that they correspond to the k^{th} degree of freedom of the $\mathbf{u}_i$ and $\mathbf{u}$ vectors, respectively.

Contribution Ratio for Strain and Stress

Once the displacement contributions $\mathbf{u}_i$ are obtained, it is possible to define the strain ε_i and stress σ_i contributions via

[7] $\varepsilon_i = \mathbf{B}\mathbf{u}_i$ and $\sigma_i = \mathbf{D}\varepsilon_i$

in which **B** and **D** are the usual strain-displacement and constitutive matrices appearing in $\varepsilon = \mathbf{B}\mathbf{u}$ and $\sigma = \mathbf{D}\varepsilon$, with ε and σ denoting total strain and stress, respectively. It should be noted that the displacements $\mathbf{u}$ and $\mathbf{u}_i$ now take on their elemental or local definitions, in which the local values are extracted from the global vectors. The contribution ratio for strain (or stress) is obtained by dividing the strain (or stress) contribution by the corresponding total value for the particular component being evaluated, i.e., symbolically written as $\mathrm{CR} = \varepsilon_i / \varepsilon$ (or $\mathrm{CR} = \sigma_i / \sigma$).

SAMPLE PROBLEM

The problem presented in this section describes elements of a sensitivity analysis completed for the design of a pavement structure. As part of the design process, different scenarios were examined to determine the effect that the introduction of a capping layer has on the displacements that are measured at the surface via falling weight deflectometer testing and on the vertical compressive strains that develop in the layer immediately beneath the subbase. When performing the analysis, a 40 kN static load was assumed to be applied uniformly over a circular area of 0.15 m radius, yielding an average pressure of 566 kPa. Four cases were considered using the parameters summarized in Table 2. The main variable was the thickness of the SCL layer, which had values of 0, 0, 30 and 45 cm for Cases 1 through 4, respectively. Owing to the elastostatic nature of the analyses, the damping ratio was set to zero.

Table 2. Standard parameters for pavement-subgrade system.

Layer	Thickness (cm)	Elastic Modulus (MPa)	Poisson's Ratio
Subgrade	300 – SCL thickness	30*	0.45
Subgrade Capping Layer	Variable	150	0.35
Subbase	30	200	0.35
Granular Base	15	250	0.35
Hot Mix	20	4000	0.35

* Elastic modulus for subgrade is 100 MPa for Case 1 only.

Figure 1 shows the pavement-subgrade system for Case 4, as well as a partial finite element mesh, which in its entirety consists of 600 nodes and 552 four-noded elements. The actual domain has a height of 3.65 m and extends to a radius of 9 m.

Three aspects of the pavement problem were of interest, namely:

- impact of subgrade modulus and presence of the SCL layer on sensitivity of surface deflections to the modulus of each layer ;
- effect of subgrade modulus and presence of the SCL layer on contribution ratios of maximum vertical strain immediately below the subbase; and
- contribution ratio distribution of vertical stress in the layers.

Surface Deflection Sensitivity

Figures 2 through 5 show the accumulated contribution ratios at various offsets for Cases 1 to 4, respectively. Comparing the plots for Cases 1 and 2, there are slight changes in the contribution ratios due to a change in subgrade modulus. While an expected increase in subgrade contribution ratio due to a decrease in its modulus is observed immediately under the load at zero offset, at larger offsets the reverse is true. Although the displacements at an offset of 1.2 m is dominated by the influence of the subgrade modulus for both cases, it appears that the effect of the base and subbase on displacements increase as the subgrade modulus decreases, which is unexpected. This is an important observation with respect to the backcalculation of in-situ moduli using FWD data, since a common analysis assumption is that one can estimate the subgrade modulus using the outside sensor measurement. While this assumption may be reasonable for some combinations of pavement-subgrade moduli, it may not be in others. The increase in contribution ratio for the upper layers as the subgrade modulus decreases may be attributed to the increase in load transfer within the upper layers via shear.

For Cases 3 and 4, which includes the SCL layer, the effect of the subgrade on measured displacements is reduced by the introduction of the additional layer. A close examination of the figures reveals that the combined contribution ratio of the SCL and subgrade is similar to that of the subgrade alone for Cases 1 and 2. In other words, elastostatic analysis of FWD data may not be able to properly discriminate between the modulus of the SCL and that of the subgrade.

Figures 6 to 9 summarize the accumulated contribution ratios that have been weighted using the total deflection at each offset for each of the cases. In other words, these figures provide the deflections corresponding to a quasi-static load. As expected, the introduction of a SCL layer reduces surface deflections. In fact, the placement of a SCL has the same effect on pavement surface deflection as increasing the subgrade modulus, which is expected. An interesting observation for all cases is the relatively uniform displacement contributions attributable to the subgrade over a radius of 1.2 m. The displacement contributions are such that most of the surface displacement close to the load is due to the stiffness properties of the engineered layers, with the foundation material stiffness dominating the deflections at the larger offsets. As indicated previously, this property is often exploited when backcalculating layer moduli using FWD data.

Contribution Ratio of Strain

We are going to take the position in this section that the capping layer acts as part of the foundation, with the pavement structure comprising the surface, base and subbase layers. For this reason the focus is on the maximum vertical strain immediately under the subbase; i.e., at a location corresponding to a fixed distance beneath the surface of the pavement where the load is applied. For Cases 1 and 2, this occurs near the top of the existing subgrade, with the focus point occurring within the SCL for Cases 3 and 4. Figures 10 and 11 show the weighted accumulated contribution ratio for the cases of no SCL and presence of SCL, respectively. As one might expect, the strain increases with a decrease in the support as a result of decreasing subgrade modulus, as observed in Figure 10. A bit surprising is the similar magnitude of strain contribution due to the modulus of the subgrade for Cases 1 and 2, even though there is a large difference in the subgrade modulus between the two cases. One must however realize that the contribution ratio for strain due to the subgrade decreases from Case 1 to 2; i.e., the effect of the upper layers becomes more important as the foundation becomes less stiff.

When introducing an SCL, the maximum vertical strain under the subbase is reduced to a level consistent with that corresponding to Case 1, which has the higher subgrade modulus. When making this statement it must be clear that this observation applies, strictly speaking, for the set of parameters chosen for the demonstration of the analysis procedure. As far as the vertical strain in the region of interest is concerned, the hot mix and SCL layer moduli play an important role in defining the magnitude of strain, with the other layers having a negligible effect.

Contribution Ratio of Stress

In the previous section, the weighted contribution ratios for vertical strain were shown only for one point. A natural extension would be to identify the variations of various contribution ratios throughout the domain. Rather than dealing with strain, we will focus for demonstration purposes on distributions corresponding to vertical stress, shown in Figure 12 for Case 1. In order to avoid numerical difficulties associated with dividing by small numbers, the contribution ratios are only identified for those elements where the magnitude of stress is greater than 1.001 times the minimum value. Since only a relatively small region in the figure is shaded, it is clear that most of the domain experiences a negligible increase in stress. For those regions which experience an increase greater than the minimum threshold value, the dark shaded areas imply a contribution ratio close to one, with the lighter shade denoting a much smaller value, usually close to zero. An important observation is that the dominance of a layer modulus on vertical stress is restricted, more or less, to the layer corresponding to the modulus in question. If we examine the contribution ratios in Figure 12 (a), we observe that most of the shaded area within the subgrade is virtually one, which implies that the elastic moduli of the upper layers have a negligible influence on the stress calculation in the subgrade. Only a small region along the centreline immediately below the subbase appears to be influenced by the modulus of the upper layers. Similar observations can be made when analysing the significance of the various elastic moduli on the contribution ratios for stresses (b), (c) and (d) of Figure 12.

CONCLUDING REMARKS

This paper has attempted to demonstrate the use of contribution ratio for analyzing the sensitivity of pavement-subgrade response to the layer moduli. It has been shown that this ratio objectively quantifies the sensitivity of pavement surface deflection to the location and stiffness of the individual pavement layers. The general methodology, presented in this paper provides pavement engineers with a tool to assist in analyzing the impact of variations in pavement layer thickness and stiffness on the overall pavement response to loading. The interpretation methodology using contribution ratio is still under development; however, it is anticipated that further refinement of the procedure and use of contribution ratio will assist in refining modulus values back-calculated from Falling Weight Deflectometer measurements.

ACKNOWLEDGEMENTS

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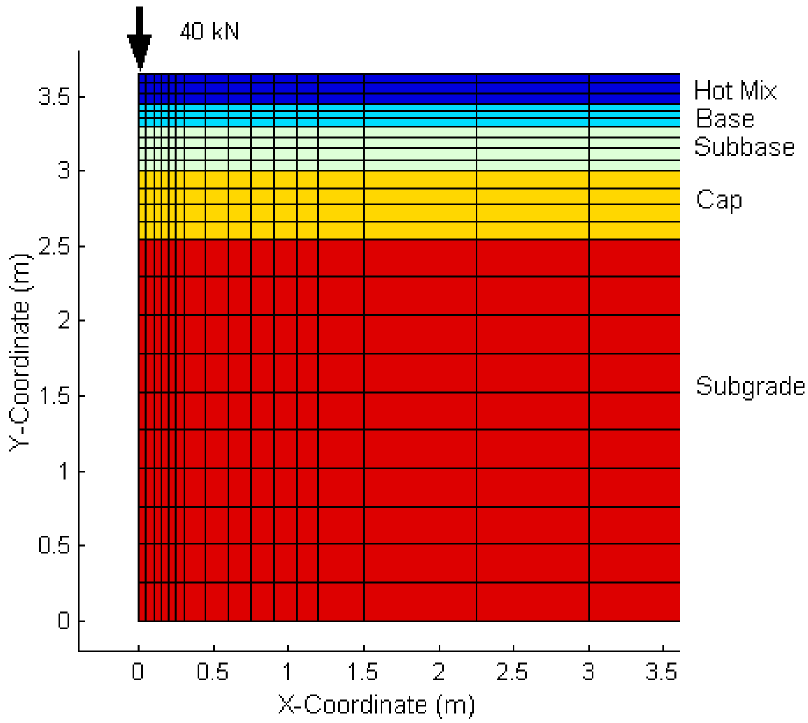

Figure 1. Finite element idealization of pavement-subgrade system.

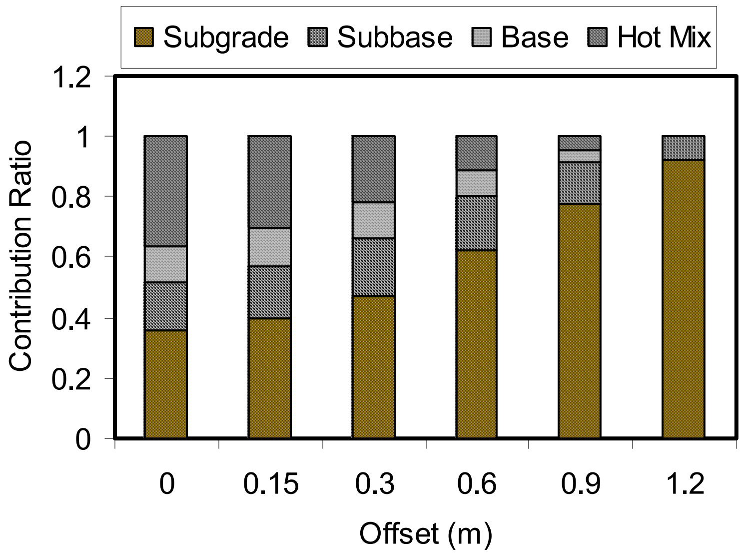

Figure 2. Contribution ratios for Case 1, 100 MPa subgrade, 30 cm subbase, 15 cm base and 20 cm HMA.

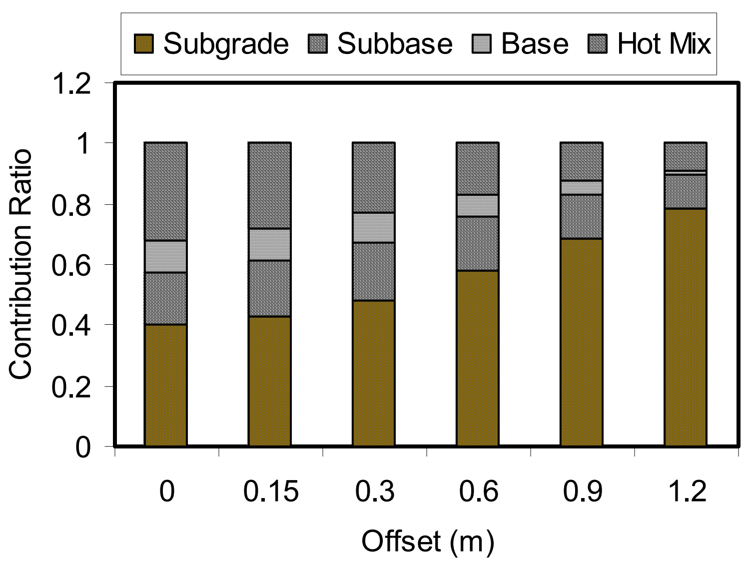

Figure 3. Contribution ratios for Case 2, 30 MPa subgrade, 30 cm subbase, 15 cm base and 20 cm HMA.

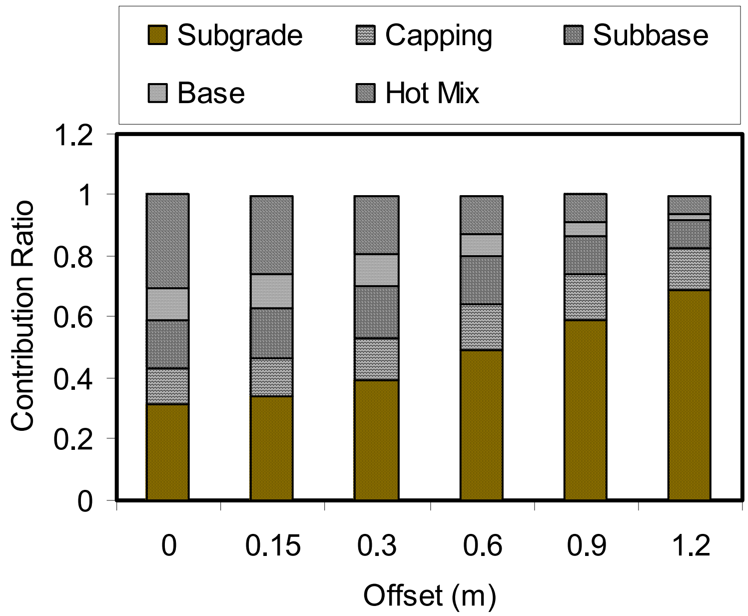

Figure 4. Contribution ratios for Case 3, 30 MPa subgrade, 30 cm SCL, 30 cm subbase, 15 cm base and 20 cm HMA.

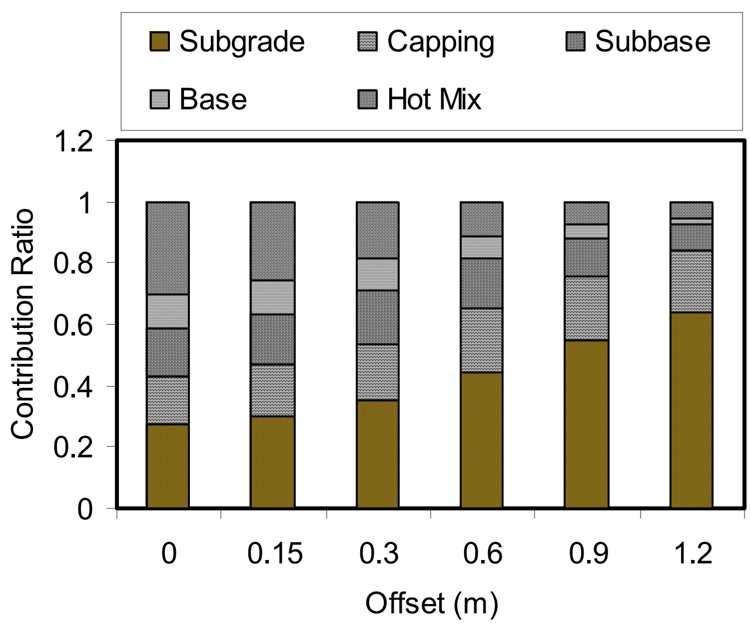

Figure 5. Contribution ratios for Case 4, 30 MPa subgrade, 45 cm SCL, 30 cm subbase, 15 cm base and 20 cm HMA.

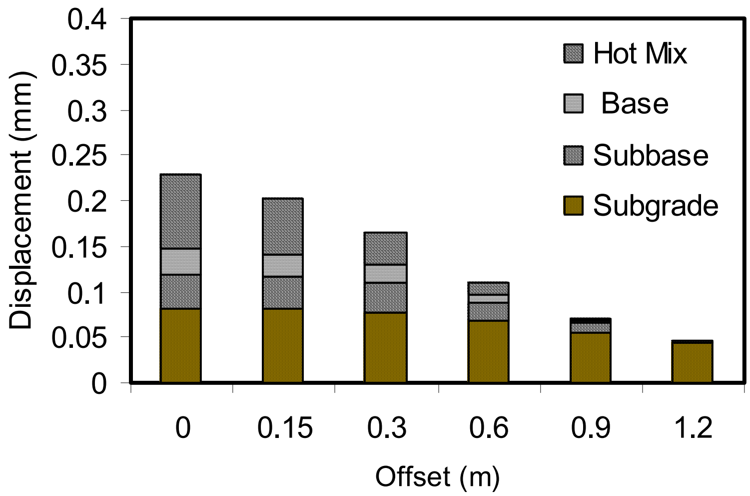

Figure 6. Displacements and weighted contribution ratios for Case 1, 100 MPa subgrade, 30 cm subbase, 15 cm base and 20 cm HMA.

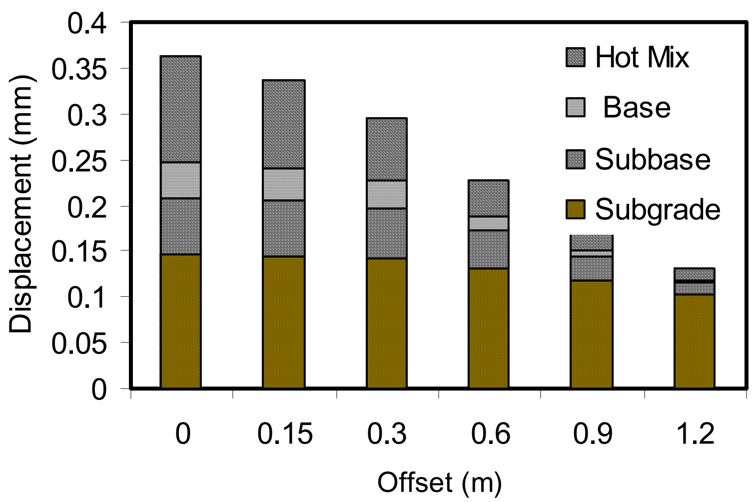

Figure 7. Displacements and weighted contribution ratios for Case 2, 30 MPa subgrade, 30 cm subbase, 15 cm base and 20 cm HMA.

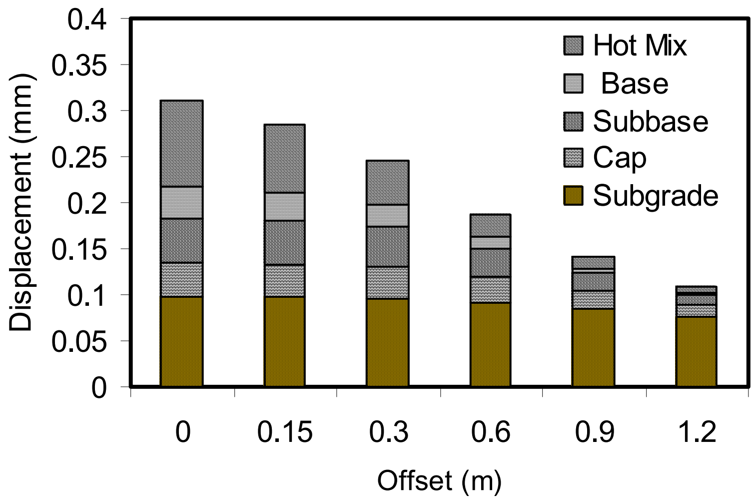

Figure 8. Displacements and weighted contribution ratios for Case 3, 30 MPa subgrade, 30 cm SCL, 30 cm subbase, 15 cm base and 20 cm HMA.

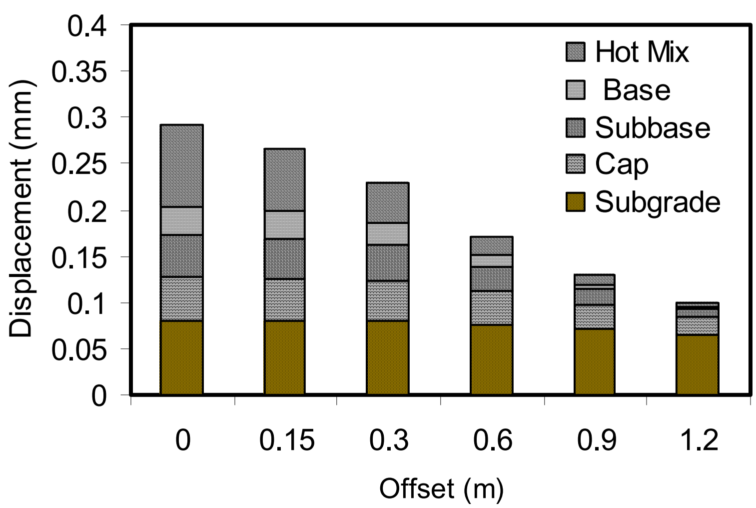

Figure 9. Displacements and weighted contribution ratios for Case 4, 30 MPa subgrade, 45 cm SCL, 30 cm subbase, 15 cm base and 20 cm HMA.

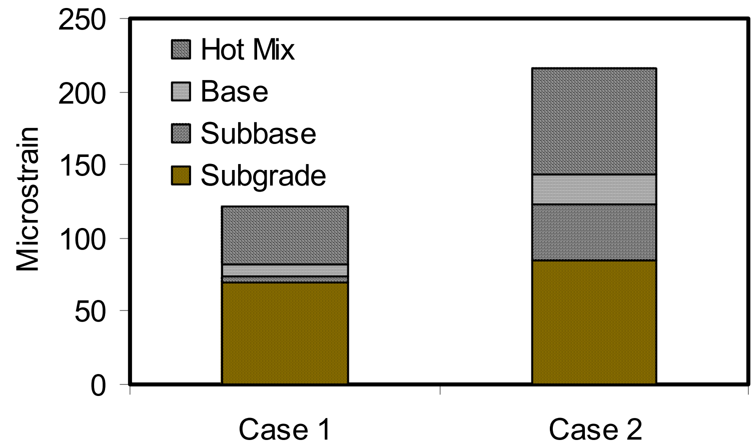

Figure 10. Maximum vertical strain and weighted contribution ratio in subgrade for Cases 1 and 2.

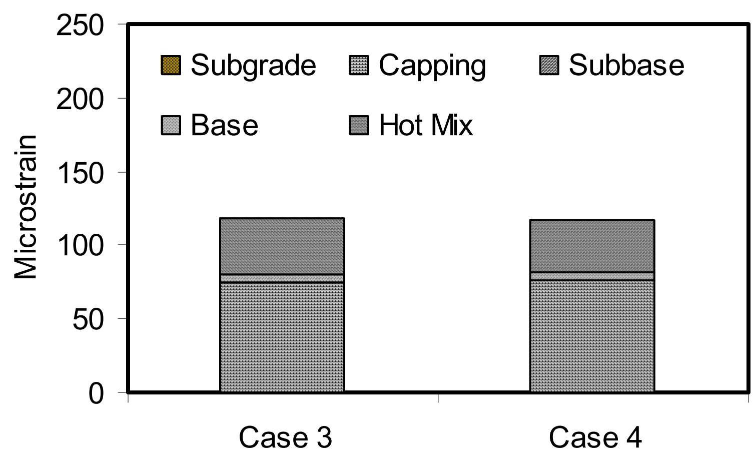

Figure 11. Maximum vertical strain and weighted contribution ratio in SCL for Cases 3 and 4.

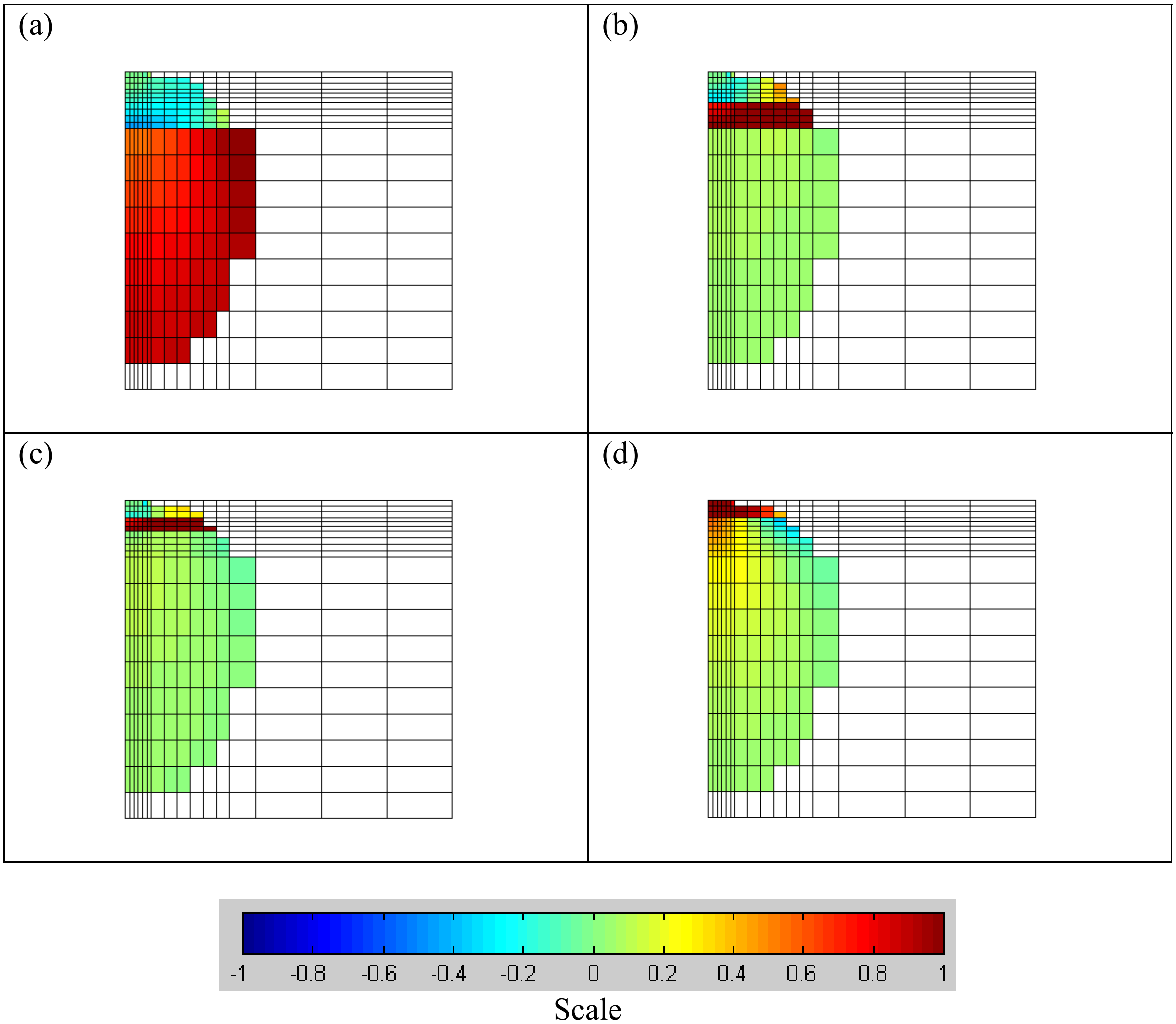

Figure 12. Contribution ratios for vertical stress for Case 1 corresponding to: (a) subgrade; (b) subbase; (c) granular base; and (d) hot mix.