Backcalculation: An Overview and Perspective

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ABSTRACT: The origins of backcalculation, including the development of equipment for deflection testing and computer programs used for deflection analysis, are reviewed. Methods and philosophies that are the basis for backcalculation are discussed. The "practical realities" of real pavements, such as bedrock, shallow water table, and stress-dependent materials, and the difficulties these pose for backcalculation are examined in detail. The importance of establishing a valid model of the pavement for backcalculation is emphasized. The effect of measurement errors, spatial and seasonal variation, and means of assessing the plausibility of the backcalculation results are discussed.

1 THE ORIGINS OF BACKCALCULATION

The procedure to determine Young's modulus of elasticity for pavement materials using measured surface deflections by working elastic layer theory "backwards" is generally called backcalculation, back calculation, back-calculation, pavement deflection analysis, or sometimes simply test and guess.

The nature of road building has evolved in recent years toward preserving and rehabilitating existing roads, more than building new ones. In many cases, pavement rehabilitation projects involve the retention of most, if not all, of the layers in the existing pavement. Thus it is useful to test the pavement in place, nondestructively, and to process the data to determine the in situ layer moduli. This process involves backcalculation.

Backcalculation is popular today because of three important advances in the field of pavement engineering.

1. The realization that strong pavements have small deflections and weak pavements have large deflections, and hence that pavement performance may be related to deflection (concept developed over the period from 1935-1960).
2. The development of mechanistic theories that relate fundamental materials properties to the stresses, strains, and deflections in a layered system (ca. 1940-1970).
3. The development of portable, accurate, and affordable instrumentation systems for measuring pavement deflections (ca. 1955-1980).

One associated event, the advent of high speed, digital computers, made it possible to accomplish the required computations in a reasonable amount of time (ca. 1960-present).

Two excellent reviews of the history of backcalculation have been written by Lytton (1989) and by Ullidtz and Coetzee (1995). This paper will supplement and expand upon the background that is provided in those two useful papers.

1.1 *Relating pavement deflection to pavement strength*

Francis (Frank) Hveem worked at the California Division of Highways (now known as CalTrans) Materials and Research Laboratory from 1930 until 1965. He served as the Director of the Laboratory from 1951 until he retired. In 1938 he began measuring transient deflections of pavements, and by 1955 he had installed 400 electronic gage units (using linear variable

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differential transformers) on 43 projects to measure deflections due to moving wheel loads (Hveem, 1955, Hveem, et al, 1962). The LVDTs were referenced to a depth of 0.91 m beneath the surface using a steel rod. The measurements were correlated to surface deflections measured with the Benkelman Beam. An extensive program of laboratory measurements of the resilient properties of materials complemented the study.

The objective of Hveem's work was to define the maximum allowable levels of deflection for use in pavement design and evaluation (Table 1). His efforts focused mainly on the peak deflection from the rolling wheel load or from the Benkelman Beam. In a 1962 paper Hveem observed:

> *Undoubtedly, the results of future deflection investigations over a variety of pavement structural sections throughout the United States will enable highway engineers to assign safe levels of deflection with reasonable certainty that they will not be overly fatigued during their design life. These deflection levels will of necessity take into account local materials, weather, mixture design and construction practices.*

Thus, in the absence of a unifying theory around which to analyze and understand pavement deflections, Hveem realized that it would be necessary to develop limiting deflection criteria for each different pavement structural section, for every materials type and for each environment. This would have been a formidable task.

Table 1. Maximum deflection for design purposes (Hveem, et al, 1962)

Type of pavement	Thickness, mm*	Maximum deflection, microns (tentative)*
Portland cement concrete	200	300
Cement treated base	150	300
Asphalt concrete	100	425
Asphalt concrete on gravel base (plant mixed)	75	500
Asphalt concrete on gravel base (plant mixed)	50	625
Asphalt concrete on gravel base (road mixed)	25	925
Surface treatment	13	1,250

* Note: The original data were converted to metric units and rounded off.

1.2 *Development of elastic layer theory and associated computer programs*

Fortunately, during the same period of time that Hveem was trying to develop allowable pavement deflection criteria, several researchers were contributing mechanistically-based theoretical tools that would enable calculation of pavement deflections. Among these were Burmister (1943, 1945, 1962), Acum and Fox (1951), Peattie (1962), and Schiffman (1962).

Theories had been published by Boussinesq in 1885 for a one-layer elastic system, and by Westergaard in 1925 for an elastic plate on a dense, liquid subgrade (i.e. no shear coupling). However, Burmister provided the first theoretical solutions for a system of two or more elastic layers, predicated on the use of Bessel's functions. This led to the landmark work by Schiffman (1962) in which he provided a general solution for an n-layer system of elastic layers.

Computer programs based on Schiffman's solution were developed in the mid-1960s by the Chevron (Michelow, 1963) and Shell (Peutz, et al, 1968) companies. The programs were called CHEV-5L and BISTRO, respectively, and they made the computational task feasible. CHEV-5L was soon replaced by a 15-layer program, and ELSYM5 allowed multiple wheel loads on the pavement. The Shell BISTRO computer program was extended and replaced by BISAR in 1973, and that program remains the standard of comparison until today.

Additional computer programs, such as WESLEA, JULEA, NELAPAV and CIRCLY, have subsequently been developed. Some offer sophisticated features.

All of the programs compute stresses, strains and displacements based on the following assumptions.

- Surface load is uniformly distributed over a circular area
- All layers are homogeneous, isotropic, and linearly elastic
- Upper layers extend horizontally to infinity
- Bottom layer is a semi-infinite half-space

Some of the programs allow multiple loads on the pavement surface, horizontal loads on the surface, and one, NELAPAV, allows all of the layers to have a stress-dependent modulus.

Computer programs based on viscoelastic layer systems have been developed by Ashton and Moavenzadeh (1967) and by Kenis (1977). A number of computer programs based on finite element analysis have also been used for pavement analysis, including SAP, ABACUS, ANSYS, ILLIPAVE, CAPA-3D, and many others. These programs involve a large number of input parameters, some of which are not readily known for pavement materials. Hence their use has mainly been limited to the pavement research community, although there has been a growing interest in the use of finite element methods among pavement engineers.

1.3 *Development of deflection testing equipment*

The relationship between pavement deflection and performance that was found by Frank Hveem led to the development of the Benkelman Beam for measuring pavement deflections. A. C. Benkelman was a pavement research engineer for the U.S. Bureau of Public Roads (now known as the Federal Highway Administration). Throughout his career, spanning from 1919 until 1962, he worked on a series of test roads. He was in charge of pavement research on the Hybla Valley (Virginia) test track in 1934, the WASHO Road Test in 1954, and the flexible pavement research at the AASHO Road Test (1959-61). He designed the Benkelman Beam for use at the WASHO Road Test (HRB, 1955), and it was used extensively at the AASHO Road Test.

The Benkelman Beam is approximately four meters long, easily portable, and it is used to measure the deflection between the two rear tires on a dump truck with a standardized axle load. The load is applied or removed slowly, over a period of several seconds, which results in deflections that include both plastic deformation and elastic deformation. On most pavements the support system for the beam is in the deflection basin. These two problems make the Benkelman Beam data particularly difficult to analyze.

The Lacroix Deflectograph is essentially a truck-mounted Benkelman Beam, which moves forward with the vehicle. Loading times are slightly faster than the manually placed Benkelman Beam.

Devices that produced vibratory oscillations on the pavement were used from the mid-1950s throughout the 1960s. The use of the Shell (Netherlands) heavy vibrator has been reported by Heukelom and Foster (1960), Heukelom and Klomp (1962), Nijboer and Metcalf (1962), and Jones, et al (1967). In this method, the modulus of elasticity of each layer can be computed from the wave velocity and wavelength for a spectrum of frequencies of oscillation. The forces within the pavement materials, however, are substantially less in this type of testing than they are under actual wheel loads.

In the early 1960s the Dynaflect pavement testing device was developed (Scrivner, et al, 1966). It produced a sinusoidal vibration at a frequency of 8 Hz, and it was outfitted with five velocity transducers (geophones), each spaced 305 mm apart, that were integrated to measure pavement deflection. The peak-to-peak load was only 4.5 kN, which was substantially less than the standard 40 kN dual-wheel load that is in common use for pavement testing. Its usefulness was enhanced, however, by being light weight and mounted on a small trailer. Some of these devices are still in use.

1.3.1 *Pulse loading equipment*

The development of equipment to provide pulse loading that would more closely approximate the timing and amplitude of a rolling wheel load began nearly simultaneously in the United States and in Europe.

From the U.S., Isada (1966) reported using a falling mass device to study the seasonal changes in the strength of flexible pavements (Figure 1). From France and Denmark, Bonitzer (1967) and Bohn, et al (1972) described the use of a falling weight deflectometer (FWD). Success with the ability to accurately measure pavement surface deflections by integrating velocity transducers, enabled the Danes to market the FWD in the late 1970's (Figure 2).

Extensive development and redesign has improved the FWD. Data collection by computer was added in 1981. Full computer control of FWD operation was added in 1982. In recent years, the ability to display and record the time history of the load pulse and deflection signals has been added, along with air and pavement temperature measurement, electronic distance measurement, and global positioning system (GPS) hardware (Figure 3).

Figure 1. Road test with an early pulse loading system (Isada, 1966)

Figure 2. FWD in use in Dubai in 1978. (photo credit: Dynatest, Inc., used with permission)

Figure 3. Modern FWD. (photo credit: Carl Bro, used with permission)

1.3.2 *Equipment manufacturers and recent developments*

I am aware of four major manufacturers of FWDs who market them around the world. They are listed in the order that they first entered the market.

- Dynatest (manufacturing facilities in Denmark and the United States)
- KUAB (Sweden)
- JILS, Foundation Mechanics, Inc. (United States)
- Carl Bro (Denmark)

In addition, Komatsu manufactures FWDS in Japan, and there are many FWDs that have been manufactured in small numbers by individual entrepreneurs. The majority of the latter are in use in Europe and South Africa.

The widespread use of FWD equipment on all six continents has provided a major impetus for the formation of FWD user's groups. One group formed in the United States and it has met annually since 1992. Another group recently formed in Europe, meeting for the first time in 2001. These meetings provide an opportunity for users to share information on data collection, data analysis, operations, and equipment maintenance. There is also a UK and Ireland FWD Users Group, but its membership is reportedly limited to FWD manufacturers and marketers.

The different manufacturers developed their equipment and software over a period of years. Until recently it has been a major problem that the data file formats are unique to each device, and several manufacturers provide more than one file format. A few of the backcalculation programs can read one of the many file formats, while other programs require an interface program to format the files, and this limits the usefulness of the data evaluation software.

Recently, a harmonized file format has been developed (AASHTO, 1998). The pavement deflection data exchange standard (PDDX) provides a uniform system for reporting FWD test results, along with sensor calibration information, site descriptions, and related information. The PDDX standard is not intended to supplant the normal file formats that each manufacturer uses.

In October 2001, at the FWD Users Group meeting in the U.S.A., all four major manufacturers announced their intention to implement the PDDX format as an output option. At the present time, both Dynatest and JILS are distributing software that provides the PDDX output. The others have indicated that they will do so soon.

PDDX output is in text, or ASCII format, which can easily be read by backcalculation programs. This should make it possible to easily analyze the data from several different brands of FWDs with a single program. Such a program is currently under development by Virginia DOT (Clarke, 2002).

1.4 *Backcalculation computer programs*

Backcalculation procedures were developed independently and simultaneously in Europe and in the United States. Among the earliest contributions for determining pavement layer moduli were nomographic solutions for two-layer systems published by Swift (1973) and Scrivner, et al (1973). Two early papers describing computer-based solutions were published by Irwin (1977) and Ullidtz (1977).

Backcalculation can be done manually by using one of the elastic layer computer programs cited in Section 1.2. However, this is a tedious, time-consuming process. So computer programs have been developed that automate the process.

A large number of computer programs for doing automated backcalculation have been written. At this time it is impossible to know about all of them, or to provide a comprehensive list. Among the more widely used programs are the following.

- ELMOD (Dynatest)
- EVERCALC (Washington State DOT)
- MODCOMP (Cornell University)
- MODULUS (Texas A&M University)
- PADAL (University of Nottingham)
- WESDEF (U.S. Army, Waterways Experiment Station)

All of the programs exist in various versions, as improved and updated editions are periodically released. Rada, et al (1992) published a comparison of the features of these and several other computer programs for backcalculation. More recently, Task Group 1, COST 336, tabulated the features of 20 different computer programs used for backcalculation (FWDUG, 2000).

Most of the automated backcalculation programs rely on an elastic layer program (ELMOD is an exception). An iterative process is required, where an initial set of layer moduli are assumed, the moduli are then used to compute surface deflections, and these are compared to the measured deflections. The assumed moduli are adjusted, and the process is repeated until the calculated deflections match the measured deflections within some specified tolerance. This process is described in more detail by Lytton (1989).

Periodically an agency attempts to do a side-by-side comparison of several backcalculation programs in order to identify the "best" one. This is a difficult task. Before making such comparisons, the agency doing so should first define its purpose (in doing backcalculation) and the evaluation criteria that it will use. Many of the programs are written for production purposes. They are intended to get to a solution reliably, and with minimum involvement of the program user. Some programs are written for use in research, and they tend to lack the features needed for production. They also usually allow and require a lot of involvement from the user.

2 THE SURFACE MODULUS CONCEPT

To gain some insight into how backcalculation works it is useful to examine the concept of the surface modulus.

The earliest contribution to elastic theory is credited to Boussinesq in 1885. He developed equations that relate the stresses, strains and displacements in a one-layer elastic material, often called a half-space. A half-space is deemed to have a horizontal surface that extends infinitely. Its depth is also infinite. It might be thought of as an infinite sphere that has been sliced in half.

The Boussinesq equations are closed-form. The relationship between the vertical deflection on the surface of the half-space, δ_Z, and elastic (Young's) modulus, E, is described by the following equations.

a. For a uniformly distributed load on the surface at r = 0:

$$\delta_Z = \frac{2P}{\pi E a}(1-\eta^2) \tag{1}$$

b. For a point load on the surface:

$$\delta_Z = \frac{P}{\pi E r}(1-\eta^2) \tag{2}$$

where P = surface load (force), r = radial distance from center of load, a = radius of loaded area, and η = Poisson's ratio.

These equations assume that the material in the half-space is homogeneous, isotropic and linearly elastic. If these assumptions are satisfied, then it is possible to calculate Young's modulus for the half-space by measuring the surface deflection due to a known load. Because the modulus is determined from a surface deflection, it is called a surface modulus, E_0.

Using independent means (viz., the BISAR program), I have calculated the surface deflections due to a 1000 kPa FWD load on a half-space of modulus 60 MPa and Poisson's ratio 0.35 (plate radius 150 mm). The resultant surface moduli, backcalculated using Equations 1 and 2, are reported in Table 2.

Equation. 1 does a fine job of determining the modulus using the center deflection (at zero radius). Equation. 2 works fairly well at larger radii, but it is not so accurate near the edge of the load plate. This is reasonable, given that Equation 2 assumes a point load, while the data are actually for a uniformly distributed load. *It is a basic principle that a mismatch between the theoretical assumptions and the actual data will almost inevitably result in an error, sometimes large and sometimes small, in the calculated modulus.*

The concept of surface modulus is less straightforward when it is applied to a pavement layer system. Deflections were calculated for a 3-layer pavement with an asphalt concrete surface layer (3,600 MPa, 250 mm thick), granular base (200 MPa, 460 mm thick), and subgrade (60 MPa). All three layers had a Poisson's ratio of 0.35. The results are reported in Table 3.

Table 2. Surface moduli, E_0, for a half space determined from surface deflections

Radius, mm	Surface deflection, microns	Surface modulus, MPa
0	4,388	60.00
200	1,798	54.92
300	1,135	58.01
600	552	59.65
1200	276	59.54
1800	181	60.49

Table 3. Surface moduli, E_0, for a 3-layer pavement determined from surface deflections

Radius, mm	Surface deflection, microns	Surface modulus, MPa
0	590	446.0
200	522	189.2
300	486	135.4
600	395	83.3
1200	266	61.9
1800	189	58.2

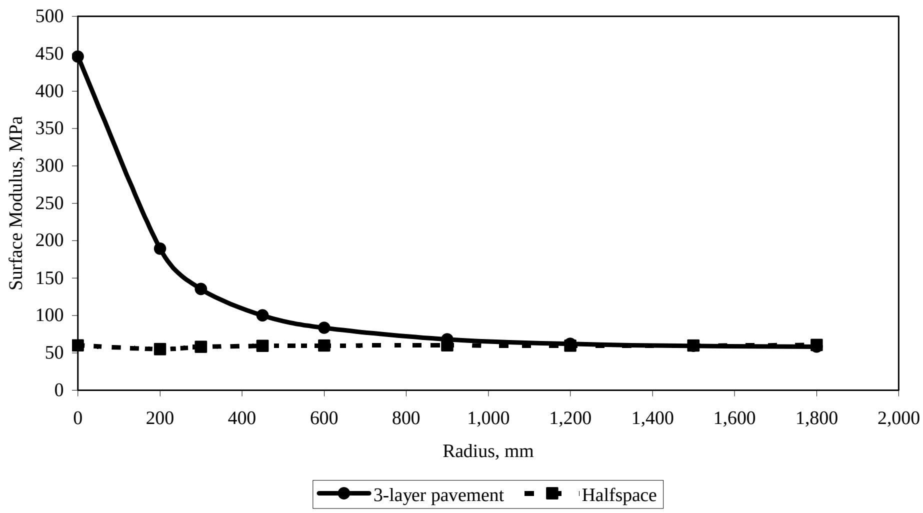

Figure 4. Surface modulus, E_0, for a 3-layer pavement and for a half-space.

The half-space equations do not work very well at all for a multilayered pavement. The center deflection is dependent on the moduli of each of the layers. So the calculated surface modulus at the center underestimates the modulus of the surface layer, and it overestimates the modulus of the subgrade. Notice, however, that the surface modulus comes close to the subgrade modulus for the deflections at large radii (Figure 4). *This illustrates another basic principle of backcalculation, namely that the outer deflections can be used to determine the moduli of the deeper layers.*

The effect of Poisson's ratio is relatively small for most values. The term $(1-\eta^2)$ goes from 0.91 to 0.88 as Poisson's ratio moves from 0.30 to 0.35. Thus a 17 percent increase in Poisson's ratio only results in a 5.4 percent decrease in $(1-\eta^2)$. *While it is important to make a good estimate of Poisson's ratio, the consequences of being slightly incorrect are not very significant.*

3 FWD DATA ERRORS

There are three main sources of errors in FWD data: seating errors, random errors, and systematic errors. Irwin, et al (1989) showed that even very small deflection errors, on the order of 2 microns or less, can have a very large effect on the backcalculated moduli.

Seating errors occur due to the rough texture and loose debris on many pavements, particularly those surfaced with asphalt concrete. Usually all that is necessary to eliminate these errors is to apply one or two drops at each new test point and discard the data. The vibrations cause the deflection sensors to become seated.

Random deflection errors have been shown to be on the order of ±2 microns (Irwin, et al, 1989). It is likely that a large part of this error is associated with the analog-to-digital conversion of the deflections. *While random errors cannot be totally eliminated, they can be reduced by taking multiple readings and averaging the result.* The error of the mean is reduced by the square root of the number of observations used in computing the mean.

Thus if four replicate FWD drops (from the same height, on the same point) are averaged, the random error would be reduced by half. Before averaging the data, however, it is important to examine it to be sure that there is no liquefaction or compaction taking place. Each reading that is included in the average must be truly random.

Systematic errors can be reduced, and possibly eliminated, through calibration. Most FWDs are specified to have an accuracy of ±2 percent or ±2 microns, whichever is larger. This specification combines the systematic error (±2 percent) and the random error. Whenever the deflections are larger than 100 microns, the systematic error may be larger than 2 microns. Since pavement deflections larger than 100 microns are quite common, *it is important to calibrate the FWDs to reduce the effect of the systematic deflection errors on the results from backcalculation.*

In the U.S.A. the Strategic Highway Research Program began efforts to calibrate FWDs in 1988 (Richter & Irwin, 1989). The calibration equipment and protocols were further refined, and four regional calibration centers were opened in 1993 (Irwin, et al, 1994). The procedure takes less than one day to perform, and it is done once per year. As a result of the calibration, the systematic error is generally reduced to 0.3 percent or less for each individual sensor, including the load cell.

4 BEDROCK AND STRESS-DEPENDENT MATERIALS

4.1 *Bedrock and related hard layer effects*

The effect of a stiff layer at a shallow depth can be quite significant. If the subgrade layer is assumed to be a semi-infinite half-space, but in reality the subgrade layer is only a few meters deep, this will cause the backcalculated moduli for the upper layers in the pavement to be very incorrect. Bedrock, stiff clay layers, and even a shallow water table can all have this effect to some degree.

How deep is infinity? This issue can be addressed by using a forward calculation program, such as BISAR, to simulate an FWD deflection basin, with various depths to a very stiff bottom layer. Then backcalculation can be used to determine the layer moduli. While there is some influence of the assumed layer thicknesses and moduli, *it will generally be the case that when the stiff bottom layer is deeper than 12 meters or so, its presence has little or no influence on the backcalculated moduli.*

In Equation 2 it can be noted that there is a relationship between the deflection, δ_Z, and the reciprocal of the radius, r, at which it occurs. This can be used to evaluate the depth to a stiff layer.

According to Equation 2, for a half-space, the deflection can be expected to be zero when the radius is infinity. To the extent that a plot of deflection versus 1/r has an intercept that is not zero, this indicates that a stiff layer may be present at shallow depth. *A presumption is made that the radial distance to the point where the deflection is zero is closely related to the depth to the stiff bottom layer.*

The data in Table 4 are the same as for Table 3 except that a stiff layer (modulus 3,000 MPa) has been added. The depth of the 60 MPa subgrade layer was fixed at 3 m. "Bedrock" then, is at 3.71 m beneath the surface of the pavement.

Table 4. Surface deflections for a four-layer linear system with bedrock at 3.71 m.

Radius, mm	a/r	Surface deflection, microns
0	---	418
200	0.750	413
300	0.500	377
600	0.250	287
1200	0.125	161
1800	0.083	88

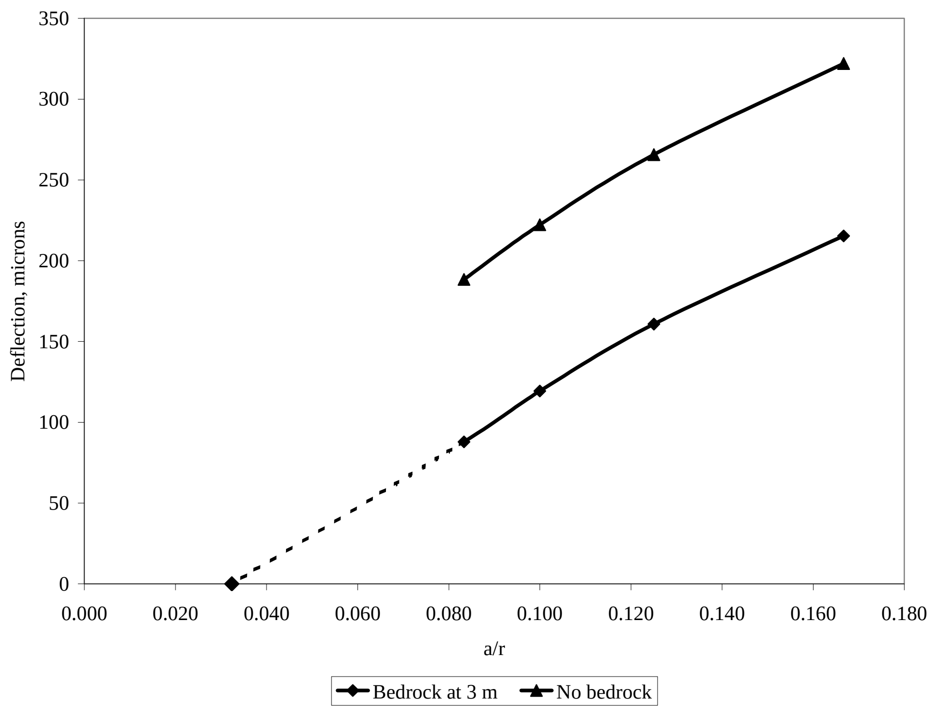

Figure 5. Prediction of depth to bedrock.

The load plate radius, a, is used to nondimensionalize the reciprocal of r. The data given in Table 4, and also that from Table 3, are plotted in Figure 5.

The data for the outer four deflections, which are all at or outside six times the plate radius ($a/r < 0.167$), are shown in Figure 5. The analysis is limited to the outer points because this is the area where the Boussinesq equation is most applicable, as is evident in Figure 4. It is clear that the data are slightly curvilinear. A linear regression through the data where bedrock is present yields an x-axis intercept of 0.03236. Hence the depth to bedrock is calculated as being 4.6 m. This value is not perfect (3.7 m is the correct answer), but it is better than no knowledge at all.

A regression through the data where no bedrock is present yields a negative intercept. This result can be interpreted to mean that there is no bedrock present (since a negative depth to bedrock is meaningless). It is useful to do a test with each regression to verify that the intercept is statistically significantly less than 12 m. If the depth is predicted to be 12 m or more, it can be considered to be infinite.

4.2 *Stress-dependent materials*

Most unbound pavement materials exhibit a modulus that is stress-dependent, or "nonlinear." Depending on the gradation and the moisture content, the modulus can either increase or decrease as the load stress increases. A material whose modulus increases with increasing stress is said to be stress hardening, while if the modulus decreases, the material is stress softening.

A linear elastic material is one that obeys Hooke's Law. It can be characterized by the constitutive model:

$$E = \text{constant} \tag{3}$$

This means that everywhere within the layer, both under the load and far away from it, the modulus is the same. Due to overburden or geostatic stresses, it is highly likely that in the subgrade layer the confining pressure increases with depth, as does the density of the material. Thus the modulus in most subgrades will increase with depth.

In a stress hardening base course or subbase, the modulus of the layer may be substantially higher immediately under the load than it is far away from the load. Thus the modulus varies horizontally within the layer, and that can pose difficulty for backcalculation if linear materials are assumed.

The upper layers in a pavement are in bending. The stress state varies vertically, as well as horizontally, and when the modulus is stress-dependent, the modulus will vary from point to point in both the vertical and horizontal directions within the layer. Assuming the material to be linear elastic totally ignores this reality.

For many years we have characterized the effect of stress using a constitutive model such as the following.

$$E = k_1 \, \text{Stress}^{k_2} \tag{4}$$

This is termed a log-log relationship because it converts to a straight line of the form $Y = mX + b$ when graphed in log-log space. The coefficient k_1 is the antilog of the intercept, b, and k_2 is the slope of the line, m.

Traditionally, the stress parameter used for sandy and gravelly materials, such as base courses, is the bulk stress.

$$\theta = \sigma_1 + \sigma_2 + \sigma_3 \tag{5}$$

For cohesive subgrade materials the deviatoric stress is used.

$$\sigma_d = \sigma_1 - \sigma_3 \tag{6}$$

In recent years the octahedral shear stress, which is a scalar invariant (it is essentially the

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad (7)$$

root-mean-square deviatoric stress), has been used for cohesive materials instead of the deviatoric stress.

To nondimensionalize the stress term, division by atmospheric pressure, p_a, is commonly done.

Equation 4 does a fine job of fitting the laboratory repeated-load triaxial data, but it does not work very well when it is applied to pavements. In the laboratory all of the applied stresses are compressive, and thus they are positive numbers. However, under the wheel load the pavement is in bending, and thus there are zones of positive stresses and zones of negative (tensile) stresses. With a negative stress term, Equation 4 will be undefined, yet it does not stand to reason that the pavement material has a modulus of zero or infinity near the load. So it must be the case that Equation 4 is not transferable from the laboratory stress environment to the pavement.

This difficulty is overcome when a semi-log constitutive model is used. This model will plot as

$$E = k_1 \exp(\text{Stress} * k_2) \qquad (8)$$

a straight line in semi-log space, where E is on the logarithmic scale. Again, k_1 is the antilog of the intercept, and k_2 is the slope of the line. This model is continuously defined, whether the stress parameter is positive or negative.

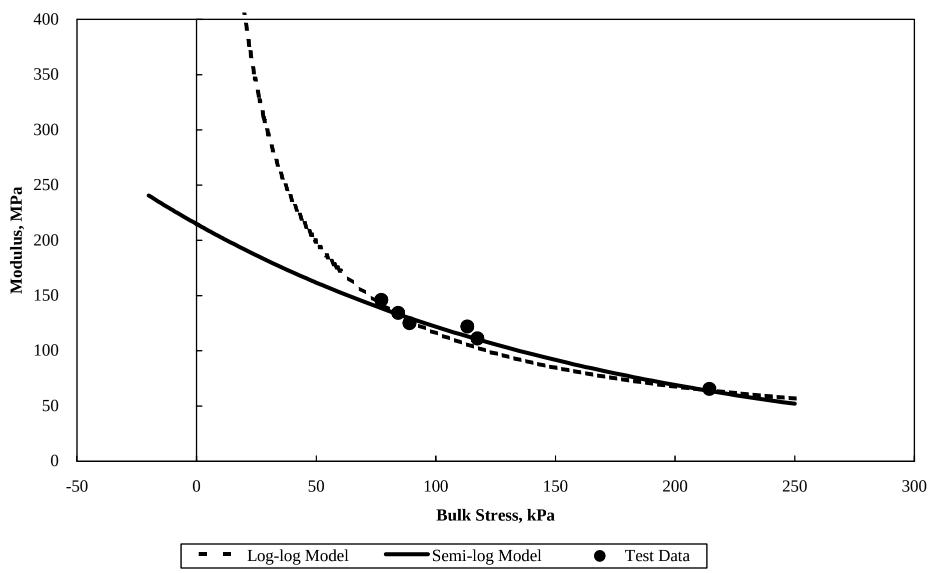

Figure 6. Comparison of log-log and semi-log constitutive models.

To illustrate this point, two models were computed for the six data points shown in Figure 6. Using a bulk stress parameter, both a log-log and a semi-log model were fitted to the data using regression methods. The results of the regressions are:

$$E = 4181\,\theta^{-0.779} \text{ (log-log model, using Equation 4)}$$

$$E = 214.8 \exp(-0.00567\,\theta) \text{ (semi-log model, using Equation 8)}$$

Both models pass through the data fairly well. The r-squared coefficient is 0.96 for the log-log model, and it is 0.95 for the semi-log model. Thus both models are highly significant. Statistically speaking, one cannot say that one model fits the data better than the other. However, it is clear in

Figure 6 that in a layer that is in bending, where the bulk stress goes from compression to tension, the log-log model would suggest that the material should become infinitely stiff. The semi-log model has no problem with the same situation.

Given the typical amount of data scatter that usually accompanies the laboratory repeated-load triaxial test, one cannot usually say which is the better model, log-log or semi-log, by inspection or by statistical analysis. However, common sense favors Equation 8. Hence I elected to use Equation 8 for stress-dependent materials in forward calculation with NELAPAV and for backcalculation with MODCOMP.

I recommend using the bulk stress parameter for base course materials and materials that are coarse-grained with little or no plasticity. For subgrade materials that have a high fines content and significant plasticity, the deviatoric stress parameter, or better yet the octahedral shear stress parameter, often works better.

A lot of research is still needed to define the "best" constitutive model for pavement materials. One model that is receiving attention in the United States was developed by Uzan (1988). It relies on two terms, one involving bulk stress and a second involving deviatoric stress. It is a log-log model, and thus it suffers from the problem of discontinuity where negative bulk stresses are involved.

At the present time, most backcalculation programs treat all layers as being linearly elastic, according to Equation 3, thus ignoring stress-dependency. A few programs allow the subgrade layer to be stress-dependent. MODCOMP allows all layers to be nonlinear. To detect the stress dependency in backcalculation is difficult because the stresses in each layer do not vary widely. This is especially true in the subgrade, where the geostatic stress is the major stress component. Backcalculation programs have great difficulty converging on the k_1 and k_2 coefficients. There is an urgent need to improve the state-of-the-art in this area.

5 THE PAVEMENT MODEL

One of the biggest challenges in doing backcalculation is in setting up the pavement model correctly. The term "pavement model" refers to the layer thicknesses and related parameters such as Poisson's ratio, that must be defined as inputs to the backcalculation computer program.

Backcalculation is constrained by the four assumptions that are listed in Section 1.2. To the extent that a pavement matches those assumptions, then the results of the backcalculation may be useful. When the pavement fails to match those assumptions, then the results may be totally useless. The objective in setting up the pavement model is to try to achieve useful results.

5.1 *Modeling thin layers - Deflection sensitivity*

For instance, consider a pavement where there is a 40 mm surface layer of hot-mix asphalt concrete over a layer of asphalt-bound base that is 150 mm thick. This covers a subbase of clean aggregate that is 250 mm thick. The subgrade is a silty sand with modest plasticity. A boring shows that free water is at 3.5 m below the surface, and bedrock is at 7.5 m beneath the surface. This appears to be a five-layer system. How should you set up the model?

To begin, it will be necessary to combine the 40 mm surface layer with the 150 mm asphalt base layer. The reason for this is the 40 mm layer is too thin to be able to backcalculate a modulus for it as a separate layer. *The basic principle is that if a layer is too thin for its modulus to have much influence on the surface deflections, then it will not be possible to use the deflections to determine the modulus of the layer.* The term "sensitivity" is used to describe the effect of a certain layer's modulus on the surface deflection. If the deflection is insensitive to the layer modulus, then any backcalculated modulus value for that layer will suffice. It may not be possible to converge on a "correct" value.

How thin is too thin? Well that depends on how deep the layer is, as well as the stiffness of the surrounding layers. It is very possible that the moduli of the two asphalt layers in the example are quite different. Differences in temperature, asphalt content, and maximum aggregate particle size could cause the layers to be very different. *If the layers are combined for backcalculation, then the modulus that is determined is a parameter of the combined layer, not a property of the material.*

5.2 *Modeling subgrades*

Let us assume that the 7 m deep subgrade layer is very homogenous. The gradation and plasticity are quite uniform throughout the layer. The entire depth would classify as being one material. But for backcalculation purposes the subgrade would need to be modeled as at least two layers.

The reason for this is because the moisture content in the subgrade is most likely not uniform. Moisture content has a big influence on modulus, particularly for cohesive materials. The upper portion of the subgrade, near the granular subbase, is most likely affected throughout the year by the weather. It may go through annual cycles of freezing and thawing, and wetting and drying.

The thickness of this upper subgrade layer may differ from one season to the next, and its depth will probably be arbitrary and not precisely known. Typical depths range from 0.5 to 1.5 meters.

The deeper portion of the subgrade is not so affected by the weather. Furthermore, it has more geostatic pressure on it. Confining stress also influences the modulus. If there is a shallow water table, such that some portion of the subgrade is saturated, this portion may also need to be modeled as a separate layer.

5.3 *Modeling bedrock*

The bedrock will need to be modeled separately. If it is very shallow, say less than 2 meters deep, it may be possible to backcalculate a modulus for the layer. If it is deeper, it will be necessary to *assign* a high, fixed value of modulus to the layer.

If the modulus of the bedrock is stiff, making it stiffer will have very little additional effect. A bedrock modulus of 3,500 MPa or so is adequate, if a high modulus must be assigned.

5.4 *The final model*

Putting it all together, we may have 5 or 6 layers in this model.

1. the combined asphalt concrete surface and asphalt concrete base (190 mm)
2. the unbound granular subbase (250 mm) – which may be very stress-dependent
3. the upper, weather affected subgrade (1000 mm) – which might be slightly stress-dependent
4. the deeper, partially saturated subgrade (roughly 2000 mm)
5. the deepest, saturated subgrade (below the water table) (4000 mm)
6. bedrock (infinitely thick, assigned a high, fixed modulus of 3,500 MPa)

It may come to pass that the fourth layer, the partially saturated subgrade, is too thin and the deflections are not sensitive to it. Whether this is so or not depends in part on the moduli of the other layers. If sensitivity is a problem, then the thickness of layer 4 will have to be combined with either layer 3 or layer 5. The choice would depend on which layer has the most similar modulus.

5.5 *A practical limitation of pavement models*

Some backcalculation programs do not allow for more than three layers plus bedrock in the pavement model. They expect there will be a surface layer, a base, a subgrade and bedrock. This can be a very significant limitation when the pavement structure is more complicated. It requires that distinctly different layers be combined, and the resultant moduli do not model the pavement very accurately.

To illustrate the effect of using different models, a forward calculation was made for the pavement model described in Section 5.1. Using bulk stress materials models in the NELAPAV 4 computer program for all layers except the asphalt and the bedrock, the pavement was divided into 20 layers. The asphalt was divided into two layers of 40 and 150 mm, the base was three equal layers, and the subgrade was 14 layers at 500 mm each. The subgrade models were adjusted in the three major zones (the weather affected upper subgrade, the middle zone, and the deeper zone below the water table). The resultant stress-dependent moduli directly beneath the load plate are shown by the gray lines in Figures 7 and 8.

A deflection basin was computed, simulating an FWD test. The advantage of using this approach is that all input parameters are controllable and known. Backcalculation was then used to determine the layer moduli according to a 4-layer and a 6-layer model. In each of these two pavement models the asphalt and base were respectively 190 and 250 mm thick. The 6-layer model divided the subgrade into three layers (1, 2, and 4 m deep), while the 4-layer model treated the subgrade as one, 7 m thick layer. Bedrock was inserted at the correct depth.

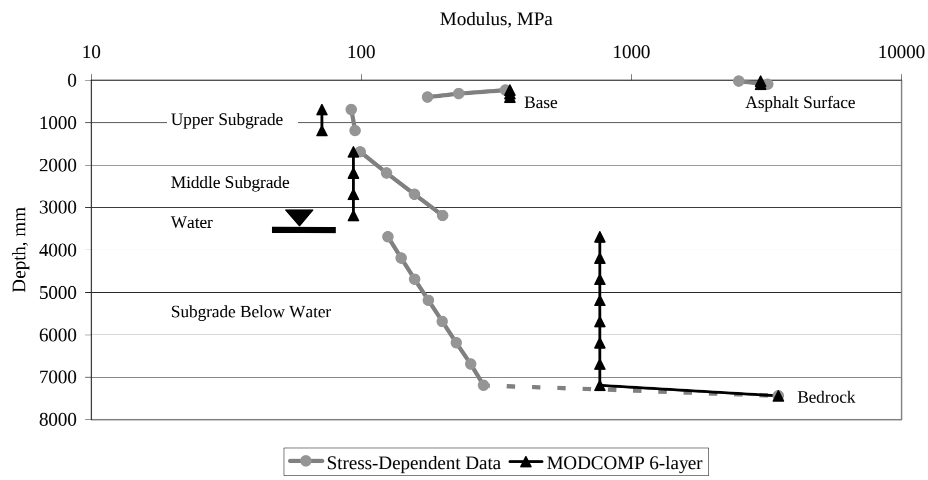

Figure 7. Nonlinear pavement system model versus 6-layer linear backcalculation

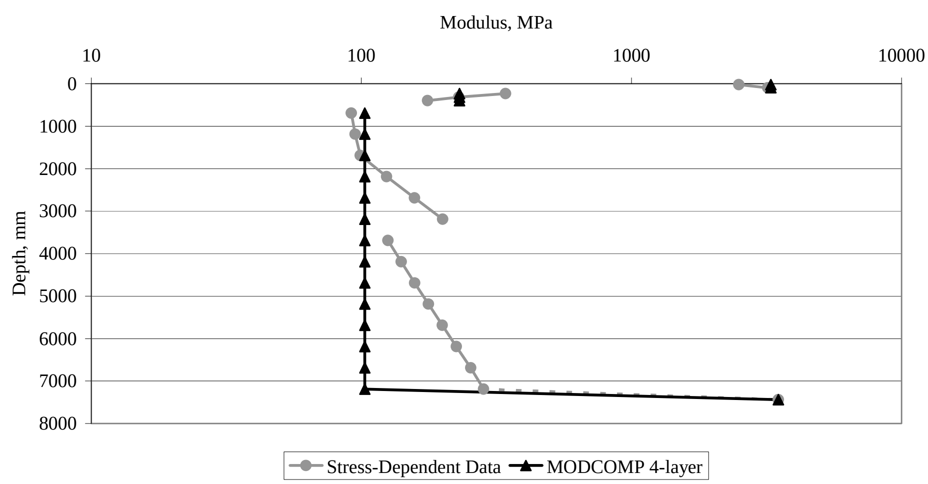

Figure 8. Nonlinear pavement system model versus 4-layer linear backcalculation

The backcalculation results are shown using the black lines and triangular symbols in Figures 7 and 8. All layers were assumed to be linear elastic (Equation 3) for the two pavement models. The root-mean-square error of fit (RMSE) for the 6-layer solution was 0.30 percent, which is excellent. The RMSE fit for the 4-layer solution was 15.49 percent, which would generally be regarded as unacceptable. MODCOMP 5 was used for the backcalculation.

It is particularly noticeable in Figure 8 that the backcalculated solution does not track the subtleties of the stress-hardening subgrade layers.

Several key stress and strain parameters were calculated for each of the three pavement models (Table 5). A 40 kN wheel load with a 690 kPa tire pressure was used for the calculations. The data shown for the 20-layer stress dependent model may be taken as the "correct" values. The results for the 6-layer and 4-layer models compare favorably, but far from exactly, to the 20-layer model.

Table 5. Comparison of stresses and strains using three pavement models.

	Forward calculated 20-layer nonlinear stress-dependent pavement model	Backcalculated 6-layer linear elastic model	Backcalculated 4-layer linear elastic model
Horizontal strain, microstrain Bottom of upper asphalt layer Depth = 40 mm	**76.8**	57.1	57.8
Horizontal strain, microstrain Bottom of lower asphalt layer Depth = 190 mm	**-180.0**	-157.1	-167.6
Vertical stress, kPa Top of base Depth = 190 mm	**106.9**	126.2	102.6
Vertical strain, microstrain Top of subgrade Depth = 440 mm	**373.3**	349.3	301.1

Note: Forward calculation used NELAPAV 4. Backcalculation used MODCOMP 5.
Positive stresses and strains are compressive.

All three models show the 40 mm upper layer of asphalt concrete to be in compression. There is fair agreement between the values of tensile strain at the bottom of the asphalt layer, 190 mm deep. This parameter would be used to predict the resistance of the asphalt surface to cracking.

The 6-layer model comes closest to predicting the vertical strain on the subgrade. These numbers are fairly low, so it is more likely that the pavement would fail due to surface cracking than due to subgrade plastic deformation.

This is only a simulation, so one should not try to infer too many conclusions from it. It can be noted that even though the pavement model was decidedly stress-dependent, a 6-layer linear model did a pretty good job of predicting the key stresses and strains. The results from the 4-layer model were not as good. The outcome of this simulation may be better than reality, because the layer thicknesses, depth to bedrock, and Poisson's ratios were known exactly here, while that would not normally be the case. Furthermore there were no random errors in the deflections.

6 ASSESSING THE VALIDITY OF BACKCALCULATED MODULI

How can you tell whether or not a particular set of layer moduli, obtained through backcalculation, is correct or not? There is not a lot of good news here. The short answer is you can never know for sure.

It is advisable to check the deflection basin fit. Parameters such as the root-mean-square error (RMSE) provide a statistic for the overall match between the measured and the backcalculated deflection basins. *When the RMS error is less than one or two percent, that is an encouraging outcome, but it does not assure that the backcalculated moduli are "correct."*

The backcalculated data in Figure 7 have a RMSE of only 0.3 percent. But inspection of the match between the 20-layer moduli and the 6-layer solution would leave many people dissatisfied with the result.

One must keep in mind that many factors can lead to erroneous results.

- There must be a good match between the assumptions that underlie backcalculation (see Section 1.2) and the realities of the pavement.
- Major cracks in the pavement, or testing near a pavement edge or joint, can cause the deflection data to depart drastically from the assumed conditions.
- Deflection data have random and systematic errors.
- It is seldom clear just how to set up the pavement model. Layer thicknesses are often not known, and subsurface layers can be overlooked. A trial and error approach is often used.
- Layer thicknesses are not uniform, nor are materials in the layers completely homogeneous.
- Some pavement layers are too thin to be backcalculated in the pavement model.
- Moisture contents and depth to hard bottom can vary widely along the road.
- Temperature gradients exist in the pavement, which can lead to modulus variation in asphalt layers and warping in concrete layers.
- Most unbound pavement materials are stress-dependent, and most backcalculation programs do not have the capability to handle that.

This is not a comprehensive list of all possible problems. But it illustrates that many difficulties do exist.

The best way to overcome the problems and to assess the validity of the backcalculated moduli is to have a thorough knowledge of the materials in the pavement. Experience will give you a basis to anticipate what moduli to expect. The inverse is also true. If you have little or no knowledge of the pavement and its materials, there is a good chance that the backcalculated moduli will not be accurate.

Many pavement evaluation projects involve testing at multiple points along the road. This can be advantageous when it comes to assessing the backcalculation results. You can compare the results from point to point to see if any "stick out" as being unreasonable. However, since subgrade moisture and depth to bedrock can vary along the road, do not be too quick to define an *unusual* result as being *unreasonable*.

The RMS error can be an effective tool. Rather than using a small RMSE to accept a set of moduli, it might be better to use a large RMSE as a way of identifying that there are problems with the pavement model. The results shown in Figures 7 and 8 give credence to this. The 4-layer solution, having a RMSE of 15.49 percent, indicates that there is a problem with the pavement model. In this case, the subgrade is a more complex system of layers than a 4-layer solution can deal with. By going to a 6-layer model, the solution is improved, as indicated by the RMSE of 0.3 percent.

Materials models also affect the validity of the result. Again, this is illustrated in Figures 7 and 8. Using linear elastic materials models cannot account for the horizontal and vertical variation in the stress-dependent layer moduli. Nevertheless, it is impossible to backcalculate a pavement model that has 20 unknown, nonlinear materials models. There simply is not enough information in nine surface deflections to yield a unique and accurate solution to such a complex problem. So compromises must be made when setting up the pavement model.

It is generally the case that the backcalculated moduli are pavement layer *parameters*, not materials *properties*. Either intentionally or unintentionally there is a discrepancy between the real pavement system and the computer pavement model. The backcalculated moduli and the real pavement resemble each other, but not perfectly. Still, the results in Table 5 show that in spite of the problems, fairly decent results are achievable.

7 SPATIAL AND SEASONAL VARIATIONS

One of the more intriguing aspects of backcalculation is its ability to unlock the mysteries of spatial and seasonal variations of pavement moduli. Until fairly recently, most pavement design methods assumed a single number that represented the layer "property" for every day of its twenty-year life. The AASHTO a-coefficient is a classic example. Modern mechanistic design methods allow us to introduce the reality that pavement layer stiffness is not constant over time or space.

7.1 *Spatial variations*

Pavements are built across cuts and fills. In most parts of the world, bedrock rises and falls beneath the pavement, and the depth to free water is often close at hand. On the macro level, the thickness of pavement layers is seldom constant. Materials, being naturally occurring substances, have different gradations, different angularity, and have received different compaction as one progresses down a road. Some areas of a road are in sunlight, and others shaded. Due to all of these factors, *there is no reason to expect pavement deflections nor backcalculated layer moduli to be constant over space*.

The effect of minor variations of layer thickness during construction, if not accounted for, can result in major errors in backcalculated layer moduli (Irwin, et al, 1989)

There is usually a big difference between the deflection test results within and between the wheelpaths. Load effects can compact and wear out pavement materials in the wheelpath. On roads with unpaved or poorly maintained shoulders, moisture can penetrate more easily under the outer wheelpath than it can under the inner wheelpath. So deflection tests in the inner and outer wheelpath may yield quite different results.

The factors noted previously tend to cause a high degree of variability of the moduli along the length of a road. This is usually dealt with statistically, by using, for instance, the 85th percentile or the 75th percentile value in subsequent analysis.

The question arises, which approach is better:

1. To do the backcalculation at all test points, then do the pavement analysis at each point, and take the 85th percentile result, or
2. To take the 85th percentile deflection basin , then do the backcalculation on this result, and then do the pavement analysis?

Clearly the second alternative involves much less work. In my experience, having specifically looked into this matter, I regretfully conclude that the first choice is the one I must recommend.

The rationale for this is straightforward. Each test point on the pavement is unique. The objective of the analysis is to find out "what to do." For instance, the project at hand may be to select an overlay thickness for a road rehabilitation project. Due to spatial variation, there is a different answer at each unique test point. By doing the analysis at each point, and afterward selecting the answer that is "right" 85 percent of the time, the spatial variation is properly accounted for (Richter & Irwin, 1988).

The location of "outliers" that have unusual requirements would be specifically known, and they should be dealt with separately.

If, however, one were to first select the 85th percentile deflection basin, this would be a basin that does not actually exist anywhere on the pavement. It would be totally fortuitous if the two approaches were to yield the same answer.

7.2 *Seasonal variations*

Temperatures and moisture conditions in the pavement vary over time. In freezing climates a pavement is strongest when the frost has penetrated beneath it. Or so it might seem. However, on a sunny day in late winter, the upper portion of base and subgrade may thaw temporarily, for just a few hours in midday, and this may in fact be the weakest and most vulnerable hours in the life of the pavement.

Pavements in areas where there is little or no freezing will also show seasonal variations in strength and deflection. Seasonal and diurnal changes in temperature will have a major effect on the modulus of an asphalt concrete layer. Changes in moisture will affect the modulus of the upper subgrade and perhaps also that of the base course.

Miner's hypothesis of cumulative fatigue damage provides a way for us to account for the long-term seasonal changes. The changes that occur within a single day, and the difference from day-to-day when one day is sunny and another is cloudy, are just annoying "noise" in the data.

When doing pavement deflection testing we seldom can choose the day. Data are obtained when they can be obtained. Yet the environmental conditions on the day of test, and on the several days before the test, will definitely have an effect on the backcalculated moduli. *There is no reason to expect pavement deflections nor backcalculated layer moduli to be constant over time.*

To take full advantage of mechanistic pavement evaluation methods, using Miner's hypothesis, the seasonal variation of pavement materials properties need to be accounted for. If FWD testing and backcalculation are used, it is vital that the pavement be tested at different times of the year to gain information about the seasonal variation. It does not suffice to send out the FWD one time, do the analysis, and assume that the tests on that one day are somehow representative of an entire year.

The backcalculated moduli are valid for the date and conditions of test, nothing more.

In the "old days" we used to dig our test pits or do our site visits to a particular road on one day of the year. From this information we would develop an overlay design that was supposedly appropriate for the next 15 years. This worked mainly because nobody checked, and if they did, nobody was held accountable. Today, with FWDs and computers available, the old ways need to be changed.

There is a lot of work that remains to be done to make it possible to correctly deal with seasonal variations in pavement moduli. In the near term, the best and only approach that makes sense is to do deflection testing at several times of the year. Then process the data.

8 SUMMARY AND CONCLUSIONS

Advances in mechanistic theory, deflection testing equipment, and computer technology over the past 50 years have made pavement deflection analysis possible. Still, backcalculation is not an easy task. It can not and should not be attempted by just anyone. It requires a good engineering background and extensive experience in order to do it well.

The key to successful backcalculation is in setting up the pavement model correctly. Limited capabilities of many backcalculation computer programs, particularly the programs that are used for production work, restrict the sophistication of the pavement model. When the layering and/or the materials models used in the pavement model are incorrect, the moduli obtained are parameters, not properties of the pavement.

Several topics in backcalculation have been noted that need further improvement.

1. Define one or more better materials models to deal with stress-dependency issues.
2. Develop more backcalculation computer programs that will allow all layers to be stress-dependent.

3. Adopt sound procedures for calibration of FWDs to reduce systematic errors.
4. Adopt a harmonized output file format, such as PDDX, and develop deflection analysis software that will read such files.
5. Develop backcalculation computer programs that assess the sensitivity of deflections to layer moduli, thus avoiding problems with "thin" layers which do not have unique moduli.
6. Develop FWD data collection strategies that will allow us to deal effectively with spatial and seasonal effects.

The field of pavement evaluation has evolved significantly since the days of the Benkelman beam. Already the technology has a certain "gee whiz" aura about it. It will be interesting to see what develops over the next 15 years or so. I am confident we will achieve new and impressive capabilities.

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